Revisiting the annihilation decay $\bar{B}_s \to \pi^+\pi^-$

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February 2, 2008

Abstract

It is very important to know the strength of annihilation contribution in B charmless nonleptonic decays. $\bar{B}_s \to \pi^+\pi^-$ process could serve a good probe of the strength. We have studied the process in QCD factorization framework. Using a gluon mass scale indicted by the studies of infrared behavior of gluon propagators to avoid enhancements in the soft end point regions, we find that the CP averaged branching ratio is about 1.24×10^{-7}, the direct CP asymmetry $C_{\pi\pi}$ is about -0.05, while the mixing-induced CP asymmetry quite large with the value $S_{\pi\pi}$=0.18. The process could be measured at LHC-b experiments in the near future and would deepen our understanding of dynamics of B charmless decays.

PACS Numbers: 13.25Hw, 12.38Bx, 12.38.Aw

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1 Introduction

Recent years many efforts have been made to understand charmless decays of B mesons, which provide good grounds to get deep insights into the flavor structure of the Standard Model (SM), the origin of CP violation, the dynamics of hadronic decays, and to search for any signals of new physics beyond the SM. Up to now, BaBar(SLAC) [1] and Belle(KEK) [2] have already accumulated large set of data and have made plenty of exciting measurements. Moreover, in the near future LHC-b experiment, the expected number of $b\bar{b}$ events produced per year is about 10^{12}, it is noted that 10% of the events would fragment to B_s mesons. This high statistics will allow studies of rare B_s decay modes, which will provide very sensitive tests of theories for B decays, electro-weak interaction models and so on.

For nonleptonic B meson decays, the most difficult aspect lies in the computation of matrix elements of the effective four-quark operators between hadron states. To deal with this, a simple and widely used approach is the so-called factorization approach(FA) [3]. In the past few years, new approaches, such as the QCD factorization(QCDF)[4] and perturbation QCD (pQCD) scheme[5] have been proposed to improve the FA on QCD grounds.

In the most cases of B meson nonleptonic decays, the annihilation contribution carries weak and strong phases different from that provided by the tree or penguin amplitudes, which is very important for studying CP-violating observables. Meanwhile, the calculation of annihilation contributions is interesting by itself, since it can help us to understand the low energy QCD dynamics and the viability of the theoretical approaches. As argued in [4], the annihilation amplitude is formally power suppressed by order Λ_{QCD}/m_b in QCDF. However, the annihilation contribution may not be small. In a recent systematic calculation of B decays[6], it is shown that the annihilation contributions could cause considerable uncertainties in their theoretical predictions, where the contributions are parameterized in term of the divergent integral $\int_0^1 \frac{dy}{y} \to X_A = (1 + \varrho_A e^{i\varphi}) \ln \frac{m_B}{\Lambda_h}$. In this paper, we argue that the strength of annihilation could be probed by measuring the interesting decay mode $\bar{B}_s \to \pi^+\pi^-$, which is a pure annihilation process. In our calculation of the scattering kernel, we will use Cornwall[7] prescription of gluon propagator with a dynamical mass to avoid enhancements in the soft end point. It is very interesting to note that recent theoretical[8] and phenomenological[9] studies are now accumulating supports for softer infrared behavior for gluon propagator. Besides serving a

probe for the annihilation, the decay has some interesting features: sizable CP violation due to both tree and penguin operators contributing, clear experimental signatures due to its two charge final states. Moreover, if few percentage of final pions are mis-identified to be muons, it would bring considerable uncertainties to the measurement of $\bar{B}_s \to \mu^+\mu^-$ at LHCb. Therefor, the decay deserves our theoretical studies using different approaches.

We have found that the CP averaged branching ratio of $\overline{B}_s \to \pi^+\pi^-$ decay is about 1.24×10^{-7}, the direct CP asymmetry $C_{\pi\pi}$ is about -0.05, while the mixing-induced CP asymmetry is as large as $S_{\pi\pi}$=0.18. Our results might be tested in the near future at LHCb.

The remaining parts of this paper are organized as follows. In the next section, we outline the necessary ingredients of the QCD factorization approach for describing the $\overline{B}_s \to \pi^+\pi^-$ decay and calculate the decay amplitude. In section 3, we give the numerical results of the CP averaged branching ratio and discuss CP asymmetries in $\overline{B}_s \to \pi^+\pi^-$ decay.

2 $\bar{B}_s \to \pi^+\pi^-$ decay in QCD factorization approach

We will start as usual from the effective Hamiltonian for the $\triangle B = 1$ transitions given by [10]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\{V_{ub}V_{us}^*[C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] - V_{tb}V_{ts}^*\sum_{i=3}^{10} C_i(\mu)O_i(\mu)\} + h.c, \tag{1}$$

where C_i are Wilson coefficients at the renormalization scale μ in the Standard Model by integrating out heavy gauge bosons and top quark fields. $O_{1,2}$ are tree operations arising from W-boson exchange and O_{3-10} are penguin operators. The values for C_i and the definition of operators O_i could be found in [10].

With the effective Hamiltonian, the amplitude for $\bar{B}_s \to \pi^+\pi^-$ in naive factorization is

$$\begin{aligned}
\mathcal{A}(\overline{B}_s \to \pi^+\pi^-) \quad &= \quad -2\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*\Big[(a_3 + \frac{3}{2}Q_u a_9)\langle\pi^+\pi^-|\overline{u}\gamma_\mu Lu|0\rangle\langle 0|\overline{s}\gamma^\mu Rb|\overline{B}_s\rangle \\
&\quad + (a_5 + \frac{3}{2}Q_u a_7)\langle\pi^+\pi^-|\overline{u}\gamma_\mu Ru|0\rangle\langle 0|\overline{s}\gamma^\mu Lb|\overline{B}_s\rangle\Big] \\
&\quad + \frac{G_F}{\sqrt{2}}V_{ub}V_{us}^* a_2\langle\pi^+\pi^-|\overline{u}\gamma_\mu Lu|0\rangle\langle 0|\overline{s}\gamma^\mu Lb|\overline{B}_s\rangle + (u \to d) \\
&= \quad -2i\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^* f_{B_s}p_B^\mu\Big[(a_3 + \frac{3}{2}Q_u a_9)\langle\pi^+\pi^-|\overline{u}\gamma_\mu Lu|0\rangle \\
&\quad + (a_5 + \frac{3}{2}Q_u a_7)\langle\pi^+\pi^-|\overline{u}\gamma_\mu Ru|0\rangle\Big] \\
&\quad + i\frac{G_F}{\sqrt{2}}V_{ub}V_{us}^* f_{B_s}p_B^\mu a_2\langle\pi^+\pi^-|\overline{u}\gamma_\mu Lu|0\rangle + (u \to d),
\end{aligned} \tag{2}$$

where $L, R = (1 \mp \gamma_5)/2$. Due to the conservation of vector current and partial conservation of axial-vector current, this amplitude will vanish in the limit $m_u, m_d \to 0$. To α_s order, the matrix $\langle \pi^+\pi^- | u \not{P}_B (1-\gamma_5) u | 0\rangle$ also vanishes due to the cancellation between the amplitudes of Fig.1 (a) and (b). So that, nonfactorizable contribution will dominate the decay, which can be obtained by calculating the amplitudes of Fig.1 (c) and (d). We consider the contribution up to the twist-3 distribution amplitude of the light mesons which is superficially suppressed by μ_π, however, μ_π is much larger than its naive scaling estimation Λ_{QCD} [4]

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = 1.5 \; GeV. \tag{3}$$

The amplitudes are calculated to be

$$\begin{aligned}
\mathcal{A}^T(\overline{B}_s \to \pi^+\pi^-) &= \frac{G_F}{\sqrt{2}} f_{B_s} f_\pi^2 \pi \alpha_s(\mu) \frac{C_F}{N_C^2} C_1 \int_0^\infty dl_+ \int_0^1 dx \int_0^1 dy \Big\{ \Phi_\pi(x)\Phi_\pi(y) \\
&\times \Big[\Big(x\Phi_+^B(l_+) + \xi \Phi_-^B(l_+) \Big) \frac{M_B^4}{D_s k_g^2} + (\xi - y)\Phi_-^B(l_+) \frac{M_B^4}{D_b k_g^2} \Big] \\
&+ \frac{\mu_\pi^2}{m_B^2} \phi_\pi(x)\phi_\pi(y) \Big[\Big(x\Phi_+^B(l_+) + y\Phi_-^B(l_+) + 3\xi \Phi_-^B(l_+) \Big) \frac{M_B^4}{D_s k_g^2} + \Big(\overline{x}\Phi_+^B(l_+) \\
&+ \overline{y}\Phi_-^B(l_+) + 3\xi\Phi_-^B(l_+) - 2\frac{m_b}{m_B}\Big(\Phi_+^B(l_+) + \Phi_-^B(l_+)\Big)\Big)\Big] \frac{M_B^4}{D_b k_g^2} \Big\},
\end{aligned} \tag{4}$$

$$\begin{aligned}
\mathcal{A}^P(\overline{B}_s \to \pi^+\pi^-) &= \frac{G_F}{\sqrt{2}} f_{B_s} f_\pi^2 \pi \alpha_s(\mu) \frac{C_F}{N_C^2} \int_0^\infty dl_+ \int_0^1 dx \int_0^1 dy \Big\{ \Phi_\pi(x)\Phi_\pi(y) \\
&\times \Big[\Big(2C_4 + \frac{C_{10}}{2} \Big) \Big((x\Phi_+^B(l_+) + \xi\Phi_-^B(l_+)) \frac{M_B^4}{D_s k_g^2} + (\xi - y)\Phi_-^B(l_+) \frac{M_B^4}{D_b k_g^2} \Big) \\
&+ \Big(2C_6 + \frac{C_8}{2} \Big) \Big(((\xi_B - x)\Phi_+^B(l_+) + \xi\Phi_-^B(l_+)) \frac{M_B^4}{D_b k_g^2} \Big) + y\Phi_-^B(l_+) \frac{M_B^4}{D_s k_g^2} \Big] \\
&+ \Big(2C_4 + 2C_6 + \frac{C_8}{2} + \frac{C_{10}}{2} \Big) \frac{\mu_\pi^2}{m_B^2} \phi_\pi(x)\phi_\pi(y) \Big[\Big(\overline{x}\Phi_+^B(l_+) \\
&+ \overline{y}\Phi_-^B(l_+) + 3\xi\Phi_-^B(l_+) - 2\frac{m_b}{m_B}\Big(\Phi_+^B(l_+) + \Phi_-^B(l_+)\Big)\Big) \frac{M_B^4}{D_b k_g^2} \\
&+ \Big(x\Phi_+^B(l_+) + y\Phi_-^B(l_+) + 3\xi\Phi_-^B(l_+) \Big) \frac{M_B^4}{D_s k_g^2} \Big] \Big\},
\end{aligned} \tag{5}$$

where $\overline{x} = 1 - x$, $\xi_B = (M_B - m_b)/M_B$, and $\xi = l_+/M_B$. $D_{b,s}$ and k_g^2 are the virtualities of b quark, s quark and gluon propagators respectively. $\Phi' s$ are the leading twist light-cone distribution amplitude(DA) of π and B mesons. $\phi_\pi(x)$ is the twist-3 DA of π meson. These distribution amplitudes can be found in Refs.[11, 12, 13, 14] which describe long-distance QCD dynamics

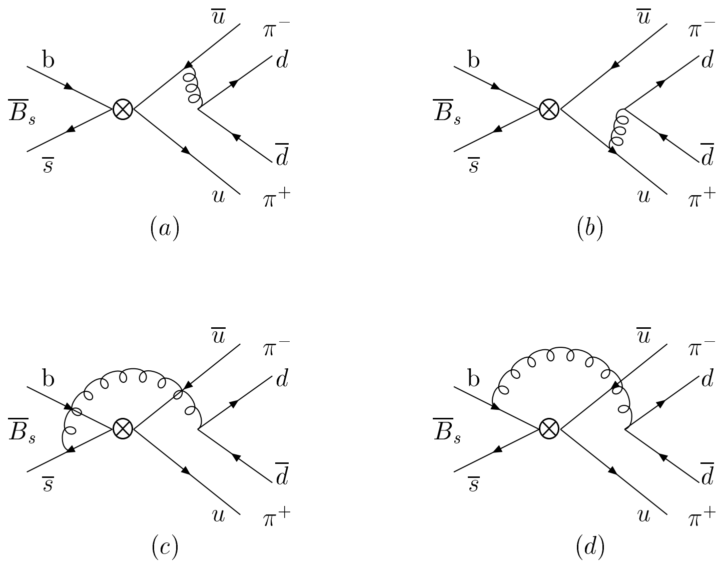


Figure 1: The annihilation diagrams for $\overline{B}_s \to \pi^+\pi^-$ decay.

of the matrix elements of quarks and mesons, which are factorized out from the perturbative short-distance interactions in the hard scattering kernels. For the distribution functions of B meson, we use the model proposed in [11]

$$\Phi_+^B(l_+) = \sqrt{\frac{2}{\pi\lambda^2}}\,\frac{l_+^2}{\lambda^2}\,\exp\Big[-\frac{l_+^2}{2\lambda^2}\Big], \tag{6}$$

$$\Phi_-^B(l_+) = \sqrt{\frac{2}{\pi\lambda^2}}\,\exp\Big[-\frac{l_+^2}{2\lambda^2}\Big]. \tag{7}$$

Now we can write the total decay amplitude

$$\mathcal{A}(\overline{B}_s \to \pi^+\pi^-) = V_{ub}V_{us}^*\mathcal{A}^T - V_{tb}V_{ts}^*\mathcal{A}^P = V_{ub}V_{us}^*\mathcal{A}^T[1 + ze^{i(\gamma+\delta)}], \tag{8}$$

where $z = |V_{tb}V_{ts}^*/V_{ub}V_{us}^*||\mathcal{A}^P/\mathcal{A}^T|$, $\gamma = \arg[V_{tb}V_{ts}^*/V_{ub}V_{us}^*]$, δ is the relative strong phase between penguin and tree contribution amplitudes, z and δ can be calculate within QCD factorization framework.

3 Numerical results and Summary

We list the parameters used in our numerical calculation [15]

$$\begin{aligned} &M_{B_s} = 5.37\ GeV, \quad m_b = 4.66\ GeV, \quad \tau_{B_s^0} = 1.461\ ps, \quad f_{B_s} = 236\ MeV, \\ &f_\pi = 130\ MeV, \qquad \bar{\rho} = 0.20, \qquad\qquad \bar{\eta} = 0.33. \end{aligned} \tag{9}$$

We set the scale in $\alpha_s(\mu)$ to be $M_{B_s}/2$ which is about the averaged virtuality of the time-like gluon. In Eq.4, 5 we meet endpoint divergences, which is the known difficulty to deal with the annihilation diagram within QCD factorization framework. Instead of the widely used treatment $\int_0^1 \frac{dy}{y} \to X_A = (1+\varrho_A e^{i\varphi}) \ln \frac{m_B}{\Lambda_h}$ in the literature[16, 17, 18], we use an effective gluon propagator [7] to avoid enhancements in the soft end point region

$$\frac{1}{k^2} \Rightarrow \frac{1}{k^2 + M_g^2(k^2)}, \qquad M_g^2(k^2) = m_g^2 \left[\frac{\ln\left(\frac{k^2+4m_g^2}{\Lambda^2}\right)}{\ln\left(\frac{4m_g^2}{\Lambda^2}\right)} \right]^{-\frac{12}{11}}. \tag{10}$$

Typically $m_g = 500 \pm 200$ MeV, $\Lambda = \Lambda_{QCD}$=250 MeV. The use of this gluon propagator is supported by lattice result [19], and field theoretical studies [8, 20] which have shown that the gluon propagator is not divergent as fast as $\frac{1}{k^2}$.

For twist-3 DA $\phi_\pi(x)$, its asymptotic form is $\phi_\pi(x) = 1$ [13] which used in [6, 18]. To further suppress endpoint contributions, we will use the recent model by Huang and Wu[14]

$$\phi_\pi(x) = \frac{A_p \beta^2}{2\pi^2} \left[1 + B_p C_2^{1/2}(1-2x) + C_p C_4^{1/2}(1-2x)\right] \exp\left[-\frac{m^2}{8\beta^2 x(1-x)}\right], \tag{11}$$

where $C_2^{1/2}(1-2x)$ and $C_4^{1/2}(1-2x)$ are Gegenbauer polynomials and other parameters could be found in [14].

Using these inputs, we get the CP averaged branching ratio of the decay

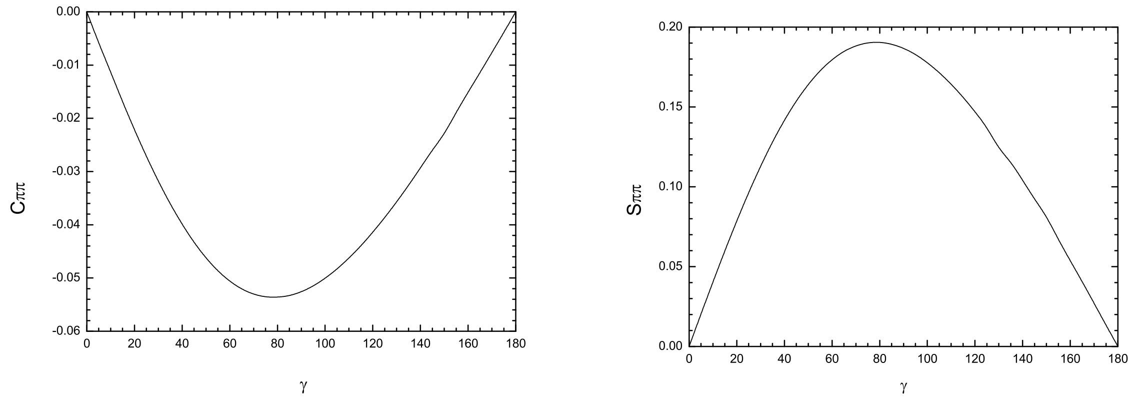
$$Br(\bar{B}_s \to \pi^+\pi^-) = (1.24 \pm 0.28) \times 10^{-7}. \tag{12}$$

The available upper limit of the decay at 90% confidence level [15] is

$$Br(\overline{B}_s \to \pi^+\pi^-) < 1.7 \times 10^{-4}. \tag{13}$$

Obviously, our result is far below this upper limit. However, our result is larger than these QCD factorization result $Br(\bar{B}_s \to \pi^+\pi^-) \simeq 2 \times 10^{-8}$ [6, 18] by using the treatment $\int_0^1 \frac{dy}{y} \to X_A = (1+\varrho_A e^{i\varphi}) \ln \frac{m_B}{\Lambda_h}$. We also note that our result may consistent with the one of Ref.[6] $Br(\bar{B}_s \to \pi^+\pi^-) = (0.024^{+0.003+0.025+0.163}_{-0.003-0.012-0.021}) \times 10^{-6}$ if the huge uncertainties are considered. In a recent study in the framework of PQCD factorization[21], the authors found $Br(\bar{B}_s \to \pi^+\pi^-) = (4.2 \pm 0.6) \times 10^{-7}$ where the end point divergence is regulated by $k_\perp^2$.

The absolute ratio between the amplitude of penguin and the tree is $z = 9.8$, and the strong phase is $\delta = 164°$. So, we can see that almost all the contribution comes from penguin. Our results for z and δ agree with the PQCD results[21].



(a) (b)

Figure 2: The direct CP violation parameter $C_{\pi\pi}$ and the mixing-induced CP violation parameter $S_{\pi\pi}$ of $\overline{B}_s \to \pi^+\pi^-$ decay as a function of weak phase γ(in degree).

Now it is time to discuss CP asymmetries of $\bar{B}_s(B_s) \to \pi^+\pi^-$. The time-dependent asymmetries are given by[22]

$$A_{CP}(t) \equiv \frac{\Gamma(\bar{B}_s(t) \to \pi^+\pi^-) - \Gamma(B_s(t) \to \pi^+\pi^-)}{\Gamma(\bar{B}_s(t) \to \pi^+\pi^-) - \Gamma(B_s(t) \to \pi^+\pi^-)} = C_{\pi\pi} \cos(\triangle mt) + S_{\pi\pi} \sin(\triangle mt), \qquad (14)$$

where $\triangle m$ is the mass difference of the two mass eigenstates of B_s meson. $C_{\pi\pi}$ and $S_{\pi\pi}$ are parameters describing the direct CP violation and the mixing-induced CP violation, respectively.

Finally, our results for direct and mixing-induced CP violations in the decay are presented as functions of weak phase γ in Fig.2.a, b respectively. For $\gamma = 60^\circ \pm 14^\circ$[15], the direct CP violation parameter $C_{\pi\pi}$ is about -0.05, the mixing-induced CP violation parameter $S_{\pi\pi}$ of the decay is as large as 0.18.

In summary, we have calculated the CP averaged branching ratio and CP asymmetries of the decay $\overline{B}_s \to \pi^+\pi^-$ within the framework of QCD factorization. We have obtained that the CP averaged branching ratio of this decay mode is of the order of 10^{-7}. The CP violations are estimated to be $C_{\pi\pi} = -0.05$, $S_{\pi\pi} = 0.18$. Compared with former studies in the same framework, we have included both the two distribution functions Φ_+^B and Φ_-^B of B_s meson. We also have used Cornwall prescription[7] for the gluon propagator with a dynamical mass to avoid enhancements in soft endpoint region. It is noted that recent studies[8, 9] have given support for Cornwall prescription, which might have many phenomenological applications in B decays. Once future measurements at LHCb in agreement with our predictions, it would indicate that

Cornwall prescription could be used in QCDF to improve it's treatment of endpoint divergences in hard-spectator scattering and annihilation topologies to enhance its power for analyzing charmless B nonleptonic decays.

Acknowledgments

This work is supported by NFSC under contract No.10305003, Henan Provincial Foundation for Prominent Young Scientists under contract No.0312001700 and in part by the project sponsored by SRF for ROCS, SEM.

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